Neutrino phenomenology and the SME

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May 23, 2021
Outline

- Introduction to neutrino oscillations
- Conventional description of neutrino oscillations
- Lorentz-violating neutrinos
- Building your own model of neutrino oscillations
- Summary
Neutrinos: some history

Beta-decay puzzle

- Observed spectrum of energies violates conservation of energy and momentum!

- **1930:** “I have done a terrible thing. I have proposed a particle that cannot be detected. It is something no theorist should ever do.”
  
  W. Pauli

- **1933:** E. Fermi formulates theory of $\beta$ decay and promotes the name *neutrino*: funny and grammatically incorrect contraction of “neutronino” (little neutron in Italian) by E. Amaldi

- **1956:** F. Reines and C. Cowan observe the first electron antineutrino (Project Poltergeist)

- **1962:** L. Lederman, M. Schwartz & J. Steinberger discover the muon neutrino

- **2000:** Tau neutrino discovered by DONUT collaboration
Neutrinos: some natural and artificial sources
Neutrinos

Properties

- Neutrinos are fundamental particles in the Standard Model (SM)
- They carry no electric charge
- They interact mostly via the weak interaction
- They come in three active flavors: $\nu_e, \nu_\mu, \nu_\tau$
- In the SM, neutrinos are massless
Neutrinos

Question #2

The Sun produces electron neutrinos $\nu_e$, whereas a nuclear reactor produces electron antineutrinos $\bar{\nu}_e$.

What is the transformation that relates neutrinos and antineutrinos?

$\nu_e \rightarrow \bar{\nu}_e$

a) C  
b) P  
c) CP  
d) CPT

Right-handed neutrinos are not observed in nature, similarly left-handed antineutrinos have never been observed.

Active neutrinos ($\nu_a L$) and active antineutrinos ($\bar{\nu}_a R$) are related by a CP transformation.
Neutrinos

Question #2

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Right-handed neutrinos are not observed in nature, similarly left-handed antineutrinos have never been observed.

\[ (\nu_e)_L \xrightarrow{C} (\bar{\nu}_e)_L \xrightarrow{P} (\bar{\nu}_e)_R \]

Active neutrinos $(\nu_a)_L$ and active antineutrinos $(\bar{\nu}_a)_R$ are related by a CP transformation.
Neutrinos

The problem of the missing neutrinos

- **1960s:** 66% of neutrinos from the Sun missing:

\[
\frac{(\Phi_{\nu_e})_{\text{exp}}}{(\Phi_{\nu_e})_{\text{th}}} \approx 0.33
\]

- **1980s:** 40% of atmospheric neutrinos missing:

\[
\frac{(\Phi_{\nu_\mu}/\Phi_{\nu_e})_{\text{exp}}}{(\Phi_{\nu_\mu}/\Phi_{\nu_e})_{\text{th}}} \approx 0.60
\]
Neutrinos: propagation of two flavors

- Neutrino eigenstates have well-defined energy
- Propagation controlled by the Hamiltonian $H$

\[
H \left( \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \right) = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \left( \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \right)
\]

If a neutrino state $|\nu_1\rangle$ is created at $t = 0$, after some time the state is

\[
|\psi(t)\rangle = e^{-iHt} |\nu_1\rangle = e^{-iE_1 t} |\nu_1\rangle
\]

The probability of measuring the state $|\nu_1\rangle$ after some time $t \simeq L$ is

\[
P_{\nu_1 \rightarrow \nu_1} = |\langle \nu_1 | \psi(t) \rangle|^2 = 1
\]

A pure $\nu_1$ beam propagates unaltered.
Neutrino mixing and oscillations of two flavors

Weakly interacting states are linear combinations of the eigenstates

\[ |\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle \]
\[ |\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle \]

If a neutrino state \( |\nu_e\rangle \) is created at \( t = 0 \), after some time the state is

\[ |\psi(t)\rangle = e^{-iHt} |\nu_e\rangle = e^{-iE_1 t} \cos \theta |\nu_1\rangle + e^{-iE_2 t} \sin \theta |\nu_2\rangle \]

A pure \( \nu_e \) beam may evolve a \( \nu_\mu \) component with time!

The probability of measuring the state \( |\nu_e\rangle \)

\[ P_{\nu_e \rightarrow \nu_e} = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta E_{21} L}{2} \right) \]

\( \Delta E_{21} \neq 0 \) indicates physics beyond the SM!
Neutrino Oscillations

Solar neutrino problem: solved

\[
\frac{(\Phi_{\nu_e})_{\text{exp}}}{(\Phi_{\nu_e})_{\text{th}}} \approx 0.33
\]

Atmospheric neutrino problem: solved

\[
\frac{(\Phi_{\nu_\mu}/\Phi_{\nu_e})_{\text{exp}}}{(\Phi_{\nu_\mu}/\Phi_{\nu_e})_{\text{th}}} \approx 0.60
\]
Conventional description: three-neutrino massive model

Phenomenological extension of the SM:

- involves three massive neutrinos

\[ h = U^\dagger \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} U \]

- energy-independent mixing: \( U(\theta_{12}, \theta_{13}, \theta_{23}, \delta) \)

- neutrinos and antineutrinos are decoupled

\[ 6 \times 6 \text{ matrix} \rightarrow \mathbf{H}^{3\nu_{\text{SM}}} = \begin{pmatrix} h & 0 \\ 0 & h^* \end{pmatrix} \]

- oscillation probability

\[ P_{\nu_a \rightarrow \nu_b} (L) = \sum_{a', b'} U^*_{a' a} U_{a' b} U_{b' a} U^*_{b' b} e^{i(E_{a'} - E_{b'}) L} \]

- energy

\[ E_{a'} = \sqrt{|p|^2 + m_{a'}^2} \approx |p| + \frac{m_{a'}^2}{2|p|} \]
Conventional description: three-neutrino massive model

- The mixing matrix is the product of three rotations:

\[
U = \begin{pmatrix}
    c_{12} & -s_{12} & 0 \\
    s_{12} & c_{12} & 0 \\
    0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    c_{13} & 0 & -s_{13} e^{-i\delta} \\
    0 & 1 & 0 \\
    s_{13} e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
    1 & 0 & 0 \\
    0 & c_{23} & -s_{23} \\
    0 & s_{23} & c_{23}
\end{pmatrix}
\]

- Relevant survival probabilities:

\[
P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \approx 1 - \sin^2 2\theta_{12} \sin^2 \left( \frac{\Delta m_{21}^2 \, L}{4E} \right) \quad \text{(LB reactor)}
\]

\[
P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 \, L}{4E} \right) \quad \text{(SB reactor)}
\]

\[
P_{\nu_\mu \rightarrow \nu_\mu} \approx 1 - \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{31}^2 \, L}{4E} \right) \quad \text{(LB accelerator, atmospheric)}
\]

- Most parameters have been measured:

\[
\Delta m_{21}^2 \simeq 7.53 \times 10^{-5} \, \text{eV}^2, \quad \sin^2 \theta_{12} \simeq 0.31, \quad \sin^2 \theta_{23} \simeq 0.55
\]

\[
\Delta m_{31}^2 \simeq 2.45 \times 10^{-3} \, \text{eV}^2, \quad \sin^2 \theta_{13} \simeq 0.02, \quad \delta \simeq 1.36\pi \quad \text{(hints of } \delta > 0)\]

PDG 2020
Conventional description: three-neutrino massive model

- energy and baseline coverage of oscillation experiments
- oscillation half-wavelengths (first oscillation maximum)

\[ \lambda_{a'b'}/2 = \pi/(E_{a'} - E_{b'}) \]

**Massive model:**

\[ \lambda_{21}/2 = 2\pi E/\Delta m_{21}^2 \]
\[ \lambda_{31}/2 = 2\pi E/\Delta m_{31}^2 \]
Conventional description: three-neutrino massive model

This model successfully describes all established oscillation data

Atmospheric neutrinos: Super-Kamiokande

Reactor antineutrinos: KamLAND

Solar neutrinos: SNO+Borexino+SK

Accelerator neutrinos: MINOS+T2K+K2K
Neutrino Anomalies

**LSND (2001)**
Evidence of antineutrino oscillations at short distances ($L \sim 30$ m)
*PRD 64, 112007 (2001)*

**Gallium anomaly (2006)**
Signal of antineutrino oscillations at short distances ($L \sim 1$ m)
*PRC 73, 045805 (2006)*

**MiniBooNE (2007)**
Oscillation signal at low energies ($E \sim 300$ MeV)
*PRL 98, 231801 (2007)*

**MiniBooNE (2010)**
Different oscillation signals for neutrinos and antineutrinos
*PRL 105, 181801 (2010)*

**Reactor anomaly (2011)**
Signal of antineutrino oscillations at short distances ($L \sim 10 - 1000$ m)
*PRD 83, 073006 (2011)*

None of these results can be accommodated by the $3\nu$SM!
Neutrino Anomalies

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\]

solar, LB reactor  \quad \text{SB reactor, LB accelerator}  \quad \text{LB accelerator, atmospheric}

\[
P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \approx 1 - \sin^2 2\theta_{12} \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E} \right) \quad (\text{LB})
\]

\[
P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) \quad (\text{SB})
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\[
P_{\nu_\mu \rightarrow \nu_\mu} \approx 1 - \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right)
\]

Question #3

Why are disappearance experiments insensitive to CP violation? (why is the CP phase $\delta$ absent in the survival probabilities $P_{\nu_x \rightarrow \nu_x}$?)
Conventional description: three-neutrino massive model

the mixing matrix is the product of three rotations

$$U = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix}$$

solar, LB reactor  \hspace{1cm}  SB reactor, LB accelerator  \hspace{1cm}  LB accelerator, atmospheric

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \approx 1 - \sin^2 2\theta_{12} \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E} \right)$$ (LB)

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right)$$ (SB)

$$P_{\nu_\mu \rightarrow \nu_\mu} \approx 1 - \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right)$$

**Question #3**

Why are disappearance experiments insensitive to CP violation?
(why is the CP phase $\delta$ absent in the survival probabilities $P_{\nu_x \rightarrow \nu_x}$?)

Lorentz invariance $\Rightarrow$ CPT invariance: $P_{\nu_x \rightarrow \nu_x}$ $\xrightarrow{\text{CPT}}$ $P_{\bar{\nu}_x \rightarrow \bar{\nu}_x} = P_{\nu_x \rightarrow \nu_x}$

Neutrino disappearance is a T-invariant process: $P_{\nu_x \rightarrow \nu_x}$ $\xrightarrow{T}$ $P_{\nu_x \rightarrow \nu_x}$

CPT and T invariance requires CP invariance $\Rightarrow$ CP violation is not observable
Conventional description: three-neutrino massive model

**Question #4**

How can we search for CP violation in neutrino oscillations? \( P_{\nu_a \rightarrow \nu_b} \)

(How can we measure the CP-violating phase \( \delta \)?)

a) Search for disappearance of antineutrinos instead of neutrinos
b) Search for oscillations between different flavors
c) Search for neutrinos oscillating into antineutrinos
d) The CP-violating phase \( \delta \) is not observable in neutrino oscillations
Question #4

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a) Search for disappearance of antineutrinos instead of neutrinos
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d) The CP-violating phase \( \delta \) is not observable in neutrino oscillations

- Lorentz invariance \( \Rightarrow \) CPT invariance: \( P_{\nu_a \rightarrow \nu_b} \xrightarrow{\text{CPT}} P_{\bar{\nu}_b \rightarrow \bar{\nu}_a} = P_{\nu_a \rightarrow \nu_b} \)
- Observation of CP violation requires a T-violating process:

\[ P_{\nu_\mu \rightarrow \nu_e} \xrightarrow{T} P_{\nu_e \rightarrow \nu_\mu} \]
Question #4

How can we search for CP violation in neutrino oscillations? $\nu_a \rightarrow \nu_b$

(How can we measure the CP-violating phase $\delta$?)

a) Search for disappearance of antineutrinos instead of neutrinos
b) **Search for oscillations between different flavors**
c) Search for neutrinos oscillating into antineutrinos
d) The CP-violating phase $\delta$ is not observable in neutrino oscillations

- Lorentz invariance $\Rightarrow$ CPT invariance: $P_{\nu_a \rightarrow \nu_b} \xrightarrow{\text{CPT}} P_{\bar{\nu}_b \rightarrow \bar{\nu}_a} = P_{\nu_a \rightarrow \nu_b}$
- Observation of CP violation requires a T-violating process:

$$P_{\nu_\mu \rightarrow \nu_e} \xrightarrow{T} P_{\nu_e \rightarrow \nu_\mu}$$
Neutrinos in the SME
General Lagrange density incorporating Lorentz and CPT violation in fermions

\[ \mathcal{L} = \frac{1}{2} \overline{\Psi}_A \left( \gamma^\mu i \partial_\mu \delta_{AB} - M_{AB} + \tilde{Q}_{AB} \right) \Psi_B + \text{h.c.}, \quad A, B \in \{e, \mu, \tau, e^C, \mu^C, \tau^C\} \]
Neutrinos in the SME

General Lagrange density incorporating Lorentz and CPT violation in fermions

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\[ A, B \in \{e, \mu, \tau, e^C, \mu^C, \tau^C\} \]

- obtain field equations
- use perturbation theory (treat Lorentz violation as a small deviation)
- decompose operators using Dirac-matrices as basis
- identify Dirac and Majorana components
- project away unphysical fields
- use perturbation theory (small neutrino masses)
- from the Schrödinger equation identify effective hamiltonian
Neutrinos in the SME

General Lagrange density incorporating Lorentz and CPT violation in fermions

\[ \mathcal{L} = \frac{1}{2} \Psi_A \left( \gamma^\mu i \partial_\mu \delta_{AB} - M_{AB} + \hat{Q}_{AB} \right) \Psi_B + \text{h.c.}, \quad A, B \in \{ e, \mu, \tau, e^C, \mu^C, \tau^C \} \]

- obtain field equations
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- from the Schrödinger equation identify effective hamiltonian

\[
H_{\text{eff}} = \begin{pmatrix}
|p| + \frac{m^2}{2|p|} & 0 & 0 \\
0 & |p| + \frac{m^2}{2|p|} & 0 \\
0 & 0 & |p| + \frac{m^2}{2|p|}
\end{pmatrix} + \frac{1}{|p|} \begin{pmatrix}
\hat{a}_{\text{eff}} - \hat{c}_{\text{eff}} & -\hat{g}_{\text{eff}} + \hat{H}_{\text{eff}} & -\hat{T}_{\text{eff}} \\
-\hat{g}^\dagger_{\text{eff}} + \hat{H}^\dagger_{\text{eff}} & -\hat{a}_{\text{eff}} - \hat{T}_{\text{eff}}
\end{pmatrix},
\]

6×6 matrix
Neutrinos in the SME

The effective hamiltonian in the SME has the form

$$H_{\text{eff}} = \left( \begin{array}{c|c}
|p| + \frac{m^2}{2|p|} & 0 \\
0 & |p| + \frac{m^2}{2|p|} \end{array} \right) + \frac{1}{|p|} \left( \begin{array}{c|c|c|c}
\hat{a}_{\text{eff}} - \hat{c}_{\text{eff}} & -\hat{g}_{\text{eff}} + \hat{H}_{\text{eff}} \\
-\hat{g}_{\text{eff}}^\dagger + \hat{H}_{\text{eff}}^\dagger & -\hat{a}_{\text{eff}}^T - \hat{c}_{\text{eff}}^T \end{array} \right),$$

6×6 matrix

Neutrino 3×3 block:

$$h'_{ab} = |p|\delta_{ab} + \frac{m^2_{ab}}{2|p|} + \sum_{djm} |p|^{d-3} Y_{jm}(\hat{p}) \left[ (a_{\text{eff}}^{(d)})_{jm} - (c_{\text{eff}}^{(d)})_{jm} \right] \quad (a, b = e, \mu, \tau)$$

Novel effects

- **unconventional energy dependence** (oscillation phase proportional to $L/E, L, LE, \ldots$)
- **direction dependence** (for $j \neq 0$)
- **local time dependence** (for Earth-based experiments and $m \neq 0$)
- **CPT violation** (differences between neutrinos and antineutrinos)
- **$\nu$-$\bar{\nu}$ mixing**
Two approaches

no Lorentz violation observed in nature
⇓
construct a general framework (SME)
⇓
perform generic searches of LV

signals of new physics (anomalous results)
⇓
build a model

- LSND
- MINOS
- **MINOS**
- IceCube
- MiniBooNE
- MINOS
- Double Chooz
- T2K
- Daya Bay
- IceCube

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<td>PRD 72, 076004</td>
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<td>PRL 101, 151601</td>
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<td>Nature Phys. 14, 961</td>
<td>2018</td>
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- **bicycle**
- **tandem**
- “**BMW**”
- **puma**
- “fried chicken bicycle”
- perturbed puma

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<td>PRD 69, 016005</td>
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<td>PRD 84, 056014</td>
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<td>CPL 29, 041402</td>
<td>2012</td>
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Sidereal variation of the oscillation probability:

\[ P_{\nu_b \rightarrow \nu_a} = (P_C)_{ab} + (P_{A_s})_{ab} \sin \Omega T + (P_{A_c})_{ab} \cos \Omega T \]
\[ + (P_{B_s})_{ab} \sin 2\Omega T + (P_{B_c})_{ab} \cos 2\Omega T + \ldots \]
Searching for Lorentz violation with neutrino oscillations

In the SME, $P^{(1)}_{\mu \tau}$ is given by [8]

$$P^{(1)}_{\mu \tau} = 2L \left( (P^{(1)}_C)_{\tau \mu} + (P^{(1)}_{A_L})_{\tau \mu} \sin \omega_T \tau \mu \right)$$

$$+ (P^{(1)}_{A_r})_{\tau \mu} \cos \omega_T \tau \mu + (P^{(1)}_{B_r})_{\tau \mu} \sin 2\omega_T \tau \mu$$

$$+ (P^{(1)}_{B_L})_{\tau \mu} \cos 2\omega_T \tau \mu \right), \tag{1}$$

where $L = 735$ km is the distance from neutrino production in the NuMI beam to the MINOS FD [2]. $T_{\theta}$ is the local sidereal time (LST) at neutrino detection, and the coefficients $(P^{(1)}_C)_{\tau \mu}, (P^{(1)}_{A_L})_{\tau \mu}, (P^{(1)}_{A_r})_{\tau \mu}, (P^{(1)}_{B_r})_{\tau \mu},$ and $(P^{(1)}_{B_L})_{\tau \mu}$ contain the LV and CPTV information.
Searching for Lorentz violation with neutrino oscillations

Double Chooz experiment
- evidence of reactor antineutrino disappearance
  \[
  P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu, \bar{\nu}_\tau) < 1
  \]
- consistent with conventional model of massive neutrinos
- confirmed by other experiments

- Alternative: antineutrinos could disappear due to oscillations into neutrinos
  New test of Lorentz invariance: neutrino-antineutrino oscillations controlled by
  \[
  (12) \tilde{g}^{\alpha\beta} \quad \text{and} \quad (3) \tilde{H}^\alpha
  \]
  \[
  P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - P(\bar{\nu}_e \rightarrow \nu_e, \nu_\mu, \nu_\tau)
  \]
Building a new model

using the SME
Building a global model

A global model must describe all established data

- disappearance of solar neutrinos
- oscillation of atmospheric neutrinos $\nu_\mu \rightarrow \nu_\tau$
- disappearance of reactor antineutrinos over hundreds of km

**Solar**

**Atmospheric**

**Reactor**

---

Borexino, PRD 82, 033006 (2010)

Super-Kamiokande, PRL 81, 1562 (1998)

KamLAND, PRL 90, 021802 (2003)
Building a global model

A global model must describe all established data

1. disappearance of solar neutrinos
2. oscillation of atmospheric neutrinos $\nu_\mu \rightarrow \nu_\tau$
3. disappearance of reactor antineutrinos over hundreds of km.
4. oscillation phase proportional to $L/E$ at low and high energies

\[ E = 2 - 9 \text{ MeV} \]

\[ E = 0.2 - 100 \text{ GeV} \]
Building a global model

$L/E$ oscillation phase arises from $H \propto E^{-1}$: positive slope in the KM plot

**Model-independent description**

**Massive model**
Building a global model

Visualizing Lorentz violation in the KM plot

- mass terms produce lines with positive slope \((H \propto E^{-1})\)
- positive powers of the energy produce lines with negative slope
- individual coefficients produce well defined lines
- combinations of coefficients produce general curves
- can Lorentz violation produce the \(L/E\) phase without masses?
Building a global model

Visualizing Lorentz violation in the KM plot

- mass terms produce lines with positive slope ($H \propto E^{-1}$)
- positive powers of the energy produce lines with negative slope
- individual coefficients produce well defined lines
- combinations of coefficients produce general curves
- can Lorentz violation produce the $L/E$ phase without masses?

YES: Lorentz-violating seesaw mechanism
Building a global model

**LV seesaw mechanism:** Bicycle model

\[
H_{\text{bicycle}} = \begin{pmatrix}
-2\hat{c}E & \frac{\ddot{a}}{\sqrt{2}} & \frac{\ddot{a}}{\sqrt{2}} \\
\frac{\ddot{a}}{\sqrt{2}} & 0 & 0 \\
\frac{\ddot{a}}{\sqrt{2}} & 0 & 0
\end{pmatrix}
\]

- very simple two-parameter model
- neutrinos are massless
- involves operators of dimensions three ($\ddot{a}$) and four ($\hat{c}$)
- hamiltonian is easily diagonalizable

\[
\begin{align*}
E_1 &= -\hat{c}E - \sqrt{(\hat{c}E)^2 + \ddot{a}^2} \\
E_2 &= 0 \\
E_3 &= -\hat{c}E + \sqrt{(\hat{c}E)^2 + \ddot{a}^2}
\end{align*}
\]
LV seesaw mechanism: Bicycle model

\[
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\end{pmatrix}
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- very simple two-parameter model
- neutrinos are massless
- involves operators of dimensions three ($\hat{a}$) and four ($\hat{c}$)
- hamiltonian is easily diagonalizable
- oscillation phase at high energy:

\[
\Delta_{32}L = -\hat{c}EL + \hat{c}EL\sqrt{1 + \frac{\hat{a}^2}{(\hat{c}E)^2}}
\approx -\hat{c}EL + \hat{c}EL + \frac{\hat{a}^2}{2\hat{c}E}L
\approx \frac{\hat{a}^2}{2\hat{c}E}L
\]
Building a global model

**LV seesaw mechanism: Bicycle model**

- very simple two-parameter model
- neutrinos are massless
- involves operators of dimensions three ($\tilde{a}$) and four ($\tilde{c}$)
- Hamiltonian is easily diagonalizable
- oscillation phase at high energy:

\[
\Delta_{32}L = -\tilde{c}EL + \tilde{c}EL \sqrt{1 + \frac{\tilde{a}^2}{(\tilde{c}E)^2}}
\]

\[
\approx -\tilde{c}EL + \tilde{c}EL + \frac{\tilde{a}^2}{2\tilde{c}E}L
\]

\[
\approx \frac{\tilde{a}^2}{2\tilde{c}E}L
\]
Neutrinos in the SME

- $L/E$ behavior can be obtained by using mass terms or the LV seesaw mechanism
- unconventional energy dependence in the SME gives us more freedom
- use of high-dimension operators could explain anomalous signals
Neutrinos in the SME: global models

- bicycle model
- tandem model
- BMW model
- puma model
- isotropic bicycle model
- perturbed puma model
- many others...

Kostelecký & Mewes, PRD 70, 031902 (2004)
Katori, Kostelecký, & Tayloe, PRD 74, 105009 (2006)
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JSD & Kostelecký, PLB 700, 25 (2011); PRD 85, 016013 (2012)
Barger, Liao, Marfatia, & Whisnant, PRD 84, 056014 (2011)
Rong & Liu, CPL 29, 041402 (2012)
Take a hamiltonian $H$

diagonalize hamiltonian: determine eigenvalues ($E_{a'}$) and eigenvectors ($\hat{v}_{a'}$)
Take a hamiltonian $H$

diagonalize hamiltonian: determine eigenvalues ($E_{a'}$) and eigenvectors ($\hat{v}_{a'}$)

- **eigenvalues are given by the solutions of the cubic equation**

$$
(E_{a'})^3 - \text{Tr}(H)(E_{a'})^2 + \frac{1}{2} \left[ (\text{Tr}(H))^2 - \text{Tr}(H^2) \right] (E_{a'}) - \text{det}(H) = 0
$$

- **trick**: set $\text{det}(H) = 0$
  - one null eigenvalue: $E_0 = 0$
  - the other two eigenvalues are solutions of a quadratic equation: $E_{\pm}$
Building a global model

1. Take a Hamiltonian $H$
2. Diagonalize Hamiltonian: determine eigenvalues ($E_{a'}$) and eigenvectors ($\hat{v}_{a'}$)
3. Construct mixing matrix
   \[ U_{a' a} = \begin{pmatrix} \hat{v}_{1}^\dagger \\ \hat{v}_{2}^\dagger \\ \hat{v}_{3}^\dagger \end{pmatrix} \]
   \[ a' = 1, 2, 3; a = e, \mu, \tau \]
4. Construct oscillation probabilities
   \[ P_{\nu_a \rightarrow \nu_b} (L) = \sum_{a', b'} U_{a' a}^* U_{a' b} U_{b' a} U_{b' b}^* e^{i(E_{a'} - E_{b'})L} \]
5. Compare with established data
The puma model

- Isotropic ($j = 0$)
- Includes nonminimal terms
- Three real parameters
- Alternative to the $3\nu$SM

Effective Hamiltonian (neutrinos)

$$H_{ab}^\nu = |p|\delta_{ab} + \frac{m_{ab}^2}{2|p|} + \sum_{djm} |p|^{d-3} Y_{jm}(\hat{p}) \left[ (a_{eff}^{(d)})_{jm}^{ab} - (c_{eff}^{(d)})_{jm}^{ab} \right]$$

$$(H_{puma})_{ab}^\nu = \frac{m_{ab}^2}{2|p|} + |p|^{p-3} \frac{(a_{eff}^{(p)})_{00}^{ab}}{\sqrt{4\pi}} - |p|^{q-3} \frac{(c_{eff}^{(q)})_{00}^{ab}}{\sqrt{4\pi}}$$
The puma model

- Isotropic \((j = 0)\)
- Includes nonminimal terms
- Three real parameters
- Alternative to the 3νSM

Effective hamiltonian (neutrinos)

\[
H_{\text{puma}} = A \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + B \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + C \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

\[A = \frac{m^2}{2E}, \quad B = \tilde{a}E^2, \quad C = \tilde{c}E^5.\]

“[this model] was discovered by a systematic hunt through the jungle of possible SME-based models.”
The puma model

Hamiltonian

\[ H_{\text{puma}} = A \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + B \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + C \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

where \[ A = \frac{m^2}{2E}, \quad B = \hat{a}E^2, \quad C = \hat{c}E^5 \]

Eigenvalues

\[ E_0 = 0 \]
\[ E_{\pm} = \frac{1}{2} \left( 3A + B + C \pm \sqrt{(A - B - C)^2 + 8(A + B)^2} \right) \]

LV seesaw mechanism at high energies

\[ E_{\pm L} \approx \frac{L}{2} \left( B + C - \sqrt{(B + C)^2 + 8B^2} \right) \]
\[ \approx \frac{L}{2} \left( C - C \sqrt{1 + \frac{8B^2}{C^2}} \right) \approx \frac{L}{2} \left( C - C - \frac{4B^2}{C} \right) = -\frac{2\hat{a}^2}{\hat{c}} \frac{L}{E} \]
The puma model

**Puma model**

**3νSM**

**LB reactor** (low energy)

\[
P_{\bar{\nu}_e \rightarrow \nu_e} \approx 1 - 0.89 \sin^2 \left( \frac{3m^2 L}{4E} \right), \quad P_{\bar{\nu}_e \rightarrow \nu_e} \approx 1 - 0.87 \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)
\]

\[
3m^2 = \Delta m^2_\odot \approx 7.53 \times 10^{-5} \text{ eV}^2
\]

**atmospheric** (high energy)

\[
P_{\nu_\mu \rightarrow \nu_\mu} \approx 1 - 1.00 \sin^2 \left( \frac{\hat{\alpha}^2 L}{\hat{c}E} \right), \quad P_{\nu_\mu \rightarrow \nu_\mu} \approx 1 - 0.99 \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right)
\]

\[
\frac{\hat{\alpha}^2}{\hat{c}} = \frac{\Delta m^2_{\text{atm}}}{4} \approx 6.13 \times 10^{-4} \text{ eV}^2
\]
Established experimental results

- Accelerator neutrinos ✓
- Atmospheric neutrinos ✓
- Reactor antineutrinos ✓
- Solar neutrinos ✓
The puma model

Established experimental results
- Accelerator neutrinos ✓
- Atmospheric neutrinos ✓
- Reactor antineutrinos ✓
- Solar neutrinos ✓

Novel features
- $\nu_\mu \rightarrow \nu_e$ oscillation signal in short-baseline experiments
- amplitude $\rightarrow 0$ for $E > 450$ MeV
  - consistent with MiniBooNE low-energy excess!
- slight but observable difference between neutrinos and antineutrinos
  - consistent with MiniBooNE neutrino-antineutrino difference!
Consistency of the puma model with established data

- **Atmospheric**
  - Plot showing the probability $P_{\nu_\mu \rightarrow \nu_\mu}$ as a function of $L/E$ (km/GeV).

- **Accelerator**
  - Plot showing the probability $P_{\nu_\mu \rightarrow \nu_\mu}$ as a function of energy $E$ (GeV).

- **Reactor**
  - Plot showing the probability $P_{\nu_e \rightarrow \bar{\nu}_e}$ as a function of $L/E$ (km/MeV).

- **Solar**
  - Plot showing the probability $(P_{ee})$ as a function of energy $E$ (MeV).
The puma model

Predictions:

- \((\sin^2 2\theta_{13})_{T2K} > (\sin^2 2\theta_{13})_{\text{MINOS}}\)

T2K, PRL 107, 041801 (2011)
MINOS, PRL 107, 181802 (2011)

Jorge S. Diaz (Indiana University)
Neutrino phenomenology and the SME
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The puma model

Predictions:

- $\left(\sin^2 2\theta_{13}\right)_{\text{T2K}} > \left(\sin^2 2\theta_{13}\right)_{\text{MINOS}}$

  ![Graph showing predictions for T2K and MINOS.]

  - T2K, PRL 107, 041801 (2011)
  - MINOS, PRL 107, 181802 (2011)

- $\left(\sin^2 2\theta_{13}\right)_{\text{reactors}} = 0$ \times

  - Daya Bay: $\sin^2 2\theta_{13} = 0.092 \pm 0.018$ PRL 108, 191802 (2012)
  - Double Chooz: $\sin^2 2\theta_{13} = 0.085 \pm 0.051$ PRL 108, 131801 (2012)
  - RENO: $\sin^2 2\theta_{13} = 0.113 \pm 0.023$ PRL 108, 171803 (2012)
Beyond neutrino oscillations

SME Neutrino Sector

flavor mixing

oscillation free

d≠3

d=3

neutrino oscillations

time of flight & weak decays

weak decays
- Sensitive to oscillation-free effects

- For beam experiments:

\[ v = 1 - \frac{m^2}{2p^2} + \sum_{dmj} (d-3)|p|^{d-4} e^{im\omega} T_{\otimes 0} N_{jm} [(a_{of})_{jm} - (c_{of})_{jm}] \]

- Neutrino velocity can depend on:
  - energy
  - sidereal time
  - direction of propagation
  - particle or antiparticles

- Physical effects
  - superluminal or subluminal propagation
  - dispersion

Effects of **dimension-three operators** (\( d = 3 \)) not observable in neutrino velocity
Lorentz-violating neutrinos: high energy

- dispersion relation for high-energy neutrinos (neglecting CPT-odd terms)

\[ E(p) = |p| - \sum_{djm} |p|^{d-3} Y_{jm}(\hat{p})(c^{(d)}_{of})_{jm} \]

- superluminal neutrinos lose energy due to Čerenkov radiation

\[ \nu \rightarrow \nu + e^- + e^+ \]

\[ i\mathcal{M} = -i\sqrt{2}G_F \frac{M_Z^2}{(k + k')^2 - M_Z^2} \bar{\nu}(p')\gamma^\alpha \nu(p) \bar{u}(k)\gamma_\alpha (2\sin^2 \theta_W - P_L)u(k') \]
Lorentz-violating neutrinos: high energy

- dispersion relation for high-energy neutrinos (neglecting CPT-odd terms)

\[ E(p) = |p| - \sum_{djm} |p|^{-3} Y_j m(\hat{p})(c_{of}^{(d)})_{jm} \]

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Lorentz-violating neutrinos: beta decay

Spectrum distortion

- generated by isotropic Lorentz violation: $(a^{(3)}_{\text{of}})_{00}$
- requires searching for deviations from conventional spectrum
- effect is maximal at a well-defined energy
- $(a^{(3)}_{\text{of}})_{00}$ also controls a new source of CP violation
- other components of $(a^{(3)}_{\text{of}})_{jm}$ lead to clear signals for decay of polarized neutrons
Lorentz-violating neutrinos: beta decay

$0\nu\beta\beta$ half-life gets modified

- neutrino phase space modifies the nuclear matrix elements
- neutrino propagator modifies the effective mass

$$\frac{1}{T_{1/2}} = G(Z, Q) |M^{0\nu}|^2 m^2$$

$$m^2 \to m^2 + m \frac{g}{R} + \left( \frac{g}{R} \right)^2$$

- neutrinoless double beta decay can occur for massless neutrinos
Established data is well described by a model of massive neutrinos although there are hints that point to physics beyond the 3\(\nu\)SM.

Simple SME-based models show that neutrino data (including some anomalies) can be described using Lorentz violation.

The SME offers an excellent playground for neutrino phenomenologists.

Many people have already built their models: build yours! (and give it a fun name)

Lorentz and CPT violation in neutrinos can be studied in oscillations, time-of-flight measurements, astrophysical dispersion, and nuclear decays.
Question #6

If the oscillation probability between neutrino flavors $\nu_a, \nu_b$ is equal to the oscillation probability between antineutrinos $\bar{\nu}_a, \bar{\nu}_b$

$$P_{\nu_b \rightarrow \nu_a} = P_{\bar{\nu}_a \rightarrow \bar{\nu}_b},$$

then CPT-violating terms are absent in the effective hamiltonian.

- True
- False
Question #6

If the oscillation probability between neutrino flavors $\nu_a, \nu_b$ is equal to the oscillation probability between antineutrinos $\bar{\nu}_a, \bar{\nu}_b$

$$P_{\nu_b \rightarrow \nu_a} = P_{\bar{\nu}_a \rightarrow \bar{\nu}_b},$$

then CPT-violating terms are absent in the effective hamiltonian.

- True
- False

CPT invariance $\Rightarrow P_{\nu_b \rightarrow \nu_a} = P_{\bar{\nu}_a \rightarrow \bar{\nu}_b}$

$$P_{\nu_b \rightarrow \nu_a} = P_{\bar{\nu}_a \rightarrow \bar{\nu}_b} \text{ does not guarantee CPT invariance}$$

Example: in the puma model $\hat{a}$ controls CPT violation; however, the oscillation probability for atmospheric neutrinos is

$$P_{\nu_\mu \rightarrow \nu_\mu} \approx 1 - \sin^2 \left( \frac{\hat{a}^2 L}{cE} \right) \quad \leftarrow \quad \text{CPT invariant}$$
Question #7

A hypothetical experiment measures the disappearance of $\nu_a$ neutrinos and $\bar{\nu}_a$ antineutrinos. Using the equations below, the fit to the data indicates the oscillation parameters shown in the figure

\[
P_{\nu_a \rightarrow \nu_a} = 1 - \sin^2 2\theta \sin^2 (1.27 \Delta m^2 L/E),
\]
\[
P_{\nu_a \rightarrow \bar{\nu}_a} = 1 - \sin^2 2\bar{\theta} \sin^2 (1.27 \Delta \bar{m}^2 L/E).
\]

How can we write the CPT-violating quantity $\Delta \bar{m}^2 - \Delta m^2$ in terms of SME coefficients?
Question #7

A hypothetical experiment measures the disappearance of $\nu_a$ neutrinos and $\bar{\nu}_a$ antineutrinos. Using the equations below, the fit to the data indicates the oscillation parameters shown in the figure

\[ P_{\nu_a \rightarrow \nu_a} = 1 - \sin^2 2\theta \sin^2 (1.27\Delta m^2 L/E) , \]
\[ P_{\bar{\nu}_a \rightarrow \bar{\nu}_a} = 1 - \sin^2 2\bar{\theta} \sin^2 (1.27\Delta \bar{m}^2 L/E) . \]

How can we write the CPT-violating quantity $\Delta \bar{m}^2 - \Delta m^2$ in terms of SME coefficients?

We can’t!

- In realistic effective field theory $\Delta \bar{m}^2 = \Delta m^2$ even in the presence of CPT violation
- If $P_{\nu_a \rightarrow \nu_a} \neq P_{\bar{\nu}_a \rightarrow \bar{\nu}_a}$, then there is at least one nonzero coefficient for CPT-odd Lorentz violation
- The plot shown assumes Lorentz and CPT invariance
- A different energy dependence is required in the data analysis
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